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Minimizing Effects of Base Excitation By Vibration Isolation

Authors:
NEIL SAWHNEY
KHUSHANT KHURANA
RANDEL PLACINO

Clients:
Prof. BAGLIONE
Prof. LUCHTENBERG
Prof. WOOTON

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Minimizing Effects of Base Excitation By Vibration Isolation

Abstract—**Vibration suppression can be achieved at the source or the receiver; however, vibration isolation, which uses a combination of springs and dampers to isolate the mass from input vibration, is the preferred technique. The significance of vibration isolation lies in its ability to improve the performance, reliability, and safety of structural and mechanical systems, as well as to reduce the transmission of noise and vibrations. Experimental modeling is essential for the visualization and testing of principles used to analyze and model vibration and vibration isolation. An accessible and intuitive experimental apparatus will be able to convey relevant engineering principles. Testing a modular system with sinusoidal base excitation displays an experimental spring constant of 39.74 N/m, compared to the theorized 39.5 N/m. The amplitude attenuation plot closely maps against theoretical graphs, demonstrated this apparatus' capabilities as a model tool for illustrating vibration isolation.**

I. Executive Summary

II. Introduction

In engineering practices, understanding the response of a structure or mechanical system to an external force is crucial in its design and usage. Vibrations in mechanical systems are often generated by various sources, such as rotating equipment, reciprocating machinery, or shock loads. This induced behavior of the system can lead to a number of undesirable effects. For example, vibration of wheels of a car due to bumps can transmit to the passengers and result in a uncomfortable ride or even damage the chassis. In fact, the "fragility" level of structures or mechanical systems - how much vibration the system can withstand - is often recorded by the International Organization of Standards (ISO) and used while designing equipment.

Suppressing these vibrations can happen at the source - redesigning equipment, adding damping, reducing forcing function - or the receiver - adding vibration absorbers, moving resonance. While the former might be the most effective but it is also difficult to accomplish without impairment of the source itself. Accordingly, vibration suppression is preferred at the receiver which is achieved through a technique called vibration isolation.

Vibration isolation, an essential technique used in the field of structural and mechanical engineering, refers to a collection of methods aimed at preventing or reducing the transmission of vibrations to a system. By isolating the mass from input vibration using combination of springs and dampers, the potential for vibration-induced damage or failure can be effectively reduced. Passive and active methods are the two primary techniques used for vibration isolation. Passive methods rely on mechanical devices such as springs, dampers, and isolators to absorb or dampen vibrations. Tuned Mass Dampers (TMD), rubber elastomer mounting, and spring isolators are examples of passive vibration isolation techniques that alter system characteristics such as number of degrees of freedom or take advantage of system response as attenuation.

Experimental modeling is essential for the visualization and testing of principles used to analyze and model vibration and vibration isolation. These experimental systems validate theoretical models and enable their application. Experimental modeling offers a wide range of direct application in design, allowing for immediate system characterization to remediate design issues.

The significance of vibration isolation lies in its ability to improve the performance, reliability, and safety of structural and mechanical systems. By reducing the impact of vibrations on these systems, the risk of damage, malfunction, and failure can be significantly reduced, leading to increased longevity and reduced maintenance cost. Additionally, vibration isolation can improve the comfort and quality of life for humans in buildings and other structures, by reducing the transmission of noise and vibrations. In summary, vibration isolation is an indispensable aspect of structural and mechanical engineering, and its importance cannot be overstated.[1]

III. Concept Generation

The most common example of vibration isolation is automotive suspension and the team decided initially to replicate a rig which demonstrates the ability of a quarter car suspension model to dampen out any vibrations from the road. This was considered a possible candidate because of the team's interest in Cooper Union Motorsports and hence it was a great possibility of using the rig to get actual data for the Formula car's suspension. However, the idea was rejected in order to demonstrate a bigger picture of vibration isolation and its ability to attenuate a certain spectrum of input frequencies. Moreover, the bigger idea of the experimental setup was to be a stand-alone project and collaboration with Cooper Union Motorsports would have hindered its independence.

Accordingly, the team decided to initiate brainstorming ideas on creating a miniature quarter car model which would not only replicate an automotive suspension but would also be able to depict its ability to attenuate. A simple drawing of the supposed testing rig is shown in figure III.1. This model consisted of 2 sets of mass, spring, and damper systems which would ideally replicate a quarter car suspension model. Since the tire's rubber behaves like a spring and damper, it was represented in the rig through the first mass, spring, and damper system. Accordingly, the second mass, spring, and damper system represented the primary mass. As shown in the figure, the system was constrained from all four sides which

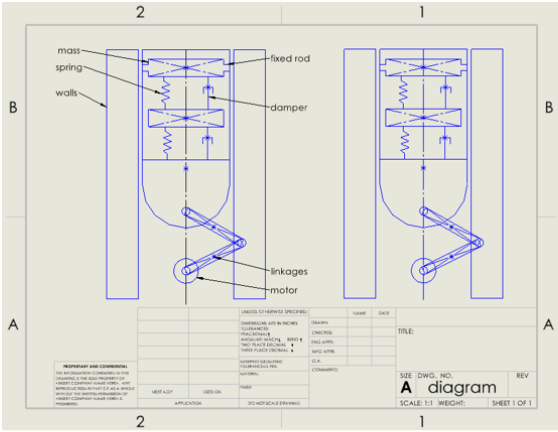

Fig. III.1. Iteration 1 : 2 sets of mass, spring, and damper systems constrained to move only vertically.

acted as the constraint. This would have not only made the procedure to replace springs, dampers, and primary mass hard but also made the fabrication process much harder.

Accordingly, the team decided to continue with only one set of mass, spring, and damper system which made the analysis much easier and also allowed for easier fabrication. Moreover, the team decided to use sinusoidal motion of the base plate as the input to the system because it makes the vibration attenuation visual and also allows the team to use their knowledge from vibrations and do a theoretical analysis. Initially, the apparatus was designed to have a two - bar linkage which connects the stepper motor to the base plate. Since the base plate is constrained to only move in vertical direction, the two - bar linkage would move the apparatus up and down. However, upon prototyping with this actuation mechanism, the team realized that this mechanism is not efficient and often becomes loose at hinges when spinning at high frequencies. Therefore, the actuation mechanism was replaced to a scotch-yoke mechanism which is explained later.

IV. Implementation of Preferred Design

After brainstorming through above mentioned design ideas, the team decided to implement a base excitation model to resemble the application of attenuation. This is done by using RC car springs and dampers to record the transmission of displacement from the excited base to the primary mass. The computer aided model of the final design idea is shown in figure IV.1.

Starting with the base mounting, it is designed to hold the stepper motor used to excite the base, threaded inserts for screwing the aluminum rods, and clamp steps to ensure the base doesn't vibrate as the motor spins. A model of the base mounting with all its specifications is shown in figure IV.2. The aluminum rods used for the structure have a outer diameter of 10mm and length of 350 mm. The setup used Nema 17 Stepper Motor with a stall torque of 59 Ncm, a rated current of 2.0 amps, and specified voltage of 12 V. The stepper motor has a 3d printed hub attached to it whose

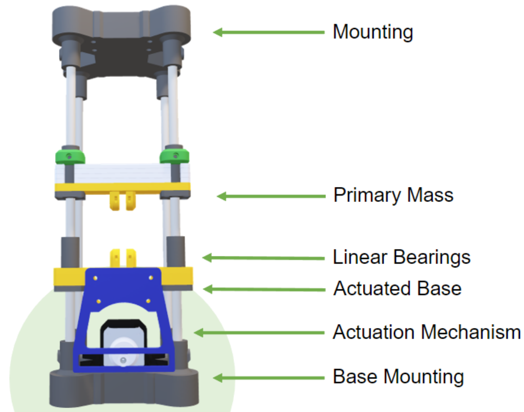

Fig. IV.1. Computer aided model of the Main assembly

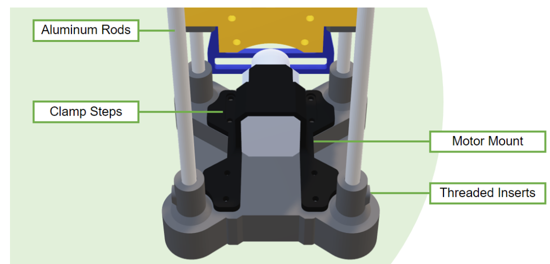

Fig. IV.2. Base mounting

amplitude can be changed. The actuation mechanism used by the apparatus is called a "scotch yoke" linkage which is a reciprocating motion mechanism converting the linear motion of a slider into rotational motion of a crank or vice versa. The apparatus uses a "deep groove ball bearings" with a inner diameter of 2 mm, outer diameter of 5 mm, and thickness of 2.3 mm. The bearing is constrained to move laterally in the blue plate shown in figure IV.3 which concurrently moves the base plate vertically. The frequency of actuation can be altered using the Arduino controlling the motor. For the primary mass,

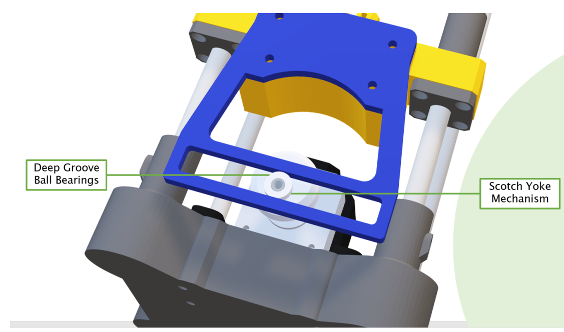

Fig. IV.3. The scotch yoke mechanism

the apparatus employed steel plates which were laser cut in a way so they can slide off the linear bearings. Each steel plate weighs 52 grams and a maximum of 4 plates can be stacked on the linear bearings. By designing the system in this way, different mass parameters can be used to run the experiment and multiple data sets can be recorded. The linear bearings reduce the friction of the system and allow the primary mass to oscillate without any external disturbances. To improve this, the bearings are supplied with sewing oil I. The linear bearings have an inner diameter of 10 mm - which matches exactly with the aluminum rods outer diameter - outer diameter of 19 mm, and a length of 55 mm.

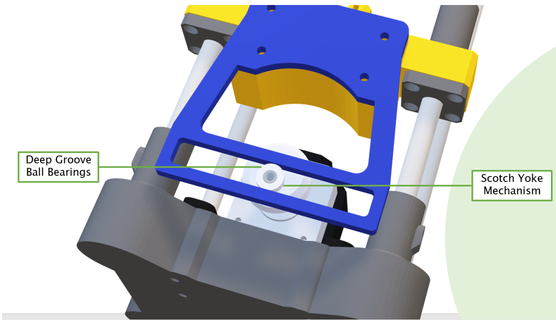

Fig. IV.4. The linear bearings hold the primary mass and actuated base, hence reducing friction in the system.

Finally, the vibration isolation component of the system is induced from a set of spring and damper attaching the excited base and primary mass. The apparatus uses RC car springs and dampers whose springs and dampers can be switched by just twisting the body. The set comes with four springs of different stiffness. The uncompressed length of the spring is 130 mm and the compressed length is 82 mm. These design specs are important for the apparatus because of packaging constraints. Accordingly, longer springs might be used if the aluminum rods are changed to a length bigger than 350 mm. Otherwise, the primary mass hits the top mounting when oscillating at resonance frequency. The placement of the spring and damper is shown in figure IV.5.

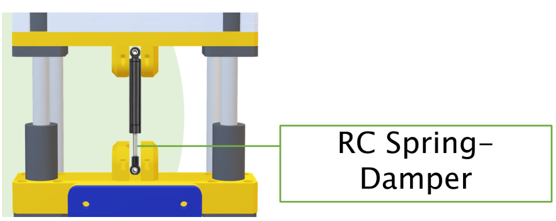

Fig. IV.5. Springs and dampers acting as the source of vibration isolation

To provide support to the entire structure, a replica of the base mounting was used to constrain the aluminum rods from top of the apparatus. The All the materials were ordered from Amazon and Adafruit, costing a total of $185.30.

V. Experimental

The study aims to reproduce the transmission of displacement from an excited base to a primary mass for various input frequencies. The input frequencies are controlled by forcing the stepper motor to step the desired times. For the motor used by the apparatus, 400 steps account for 1 revolution. Accordingly,the rotational speed of the motor which equates to the input frequency of the sinusoidal input can be altered. The workbench consists of the apparatus, Arduino, computer, and a power supply to operate the stepper motor.

A. Apparatus

The experiment requires the construction and use of a novel apparatus for demonstration of vibration isolation. This system consists of units of removable steel masses and a base plate, coupled via a spring-damper. This system relies on sinusoidal input base-excitation, driven by a stepper motor. The system is restrained to vertical motion by two adjacent, linear rods, and further supported by two linear rods in the rear. The

VI. Instrumentation

A. Distance Measurement

The output amplitude of the mass was measured using a ruler and a visual analysis software known as tracker. A set of three maximums were found for each frequency from the data. These were then averaged and plotted vs r (ω/ω_n) as shown in figure VII.1

The tracker data took longer to complete but ultimately yielded more accurate results since the high frequencies made measuring by hand mostly guess work. The tracker data lost its accuracy at high frequencies due to motion blur. Recommendations for achieving better results with this method are further explained in section XI.

B. Procedure

The experiment began by preparing the workbench and wiring up the stepper motor, motor driver (TMC2209), and Arduino. The step, enable, and direction pins were plugged into the respective pin slots of the Arduino - as mentioned the code shown in appendix A. Following this the motor driver is wired according to the schematic shown in figure VI.1 Following this,

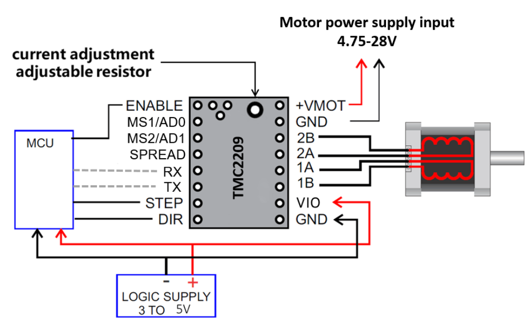

Fig. VI.1. The wiring of the motor driver to control the stepper motor.

the spring of lowest stiffness is chosen and inserted into the spring and damper housing. Once the apparatus is powered up, the Arduino is connected to the computer to use the code A and alter the frequency by interfacing through Arduino IDE. Upon running the program, the motor is operated between the

TABLE I. BILL OF MATERIALS

Item	Company	Description	Brand name	Part number	Price	Qty	Cost	For use
Ball bearings	Amazon	OD - 5mm, ID - 2mm, 2.3mm bore	Uxcell	682ZZ	$11.49	1	$11.49	Sliding mechanism
Stepper motor	Amazon	Nema 17 Stepper Motor Bipolar	Stepperonline	NEMA 17	$13.99	1	$13.99	Actuation
Linear ball bearings	Amazon	10mm Square Flange	HiPicco	LMK10LUU	$18.99	1	$18.99	Oscillation
Aluminum rods	Amazon	350mm length, 10mm diameter	VICHSAMWY	N/A	$14.99	2	$29.98	Structure
Arduino	Amazon	Controller Board ATmega328P	ELEGOO	UNO R3	$15.99	1	$15.99	Data collection
Springs & dampers	Amazon	Uncompressed = 120mm, 1/10 RC	RCLIONS	n/a	$23.99	1	$23.99	Vibration isolation
Time of flight sensor	Amazon	Distance measurement	Adafruit	VL53L4CD	$14.95	2	$29.90	Output amplitude
Sewing Oil	Amazon	Universal Sewing Machine Oil	Universal	n/a	$6.99	1	$6.99	Lubrication
Power Supply	Amazon	12 DC Volt, 2.5 Amp	ABLEGRID	n/a	$9.99	1	$9.99	Motor power supply
Hardware	Amazon	Kadrick Assorted Set	Kadrick	N/A	$23.99	1	$23.99	Assembly

frequency zone of 0 - 2.9 Hz at an interval of 0.1 Hz. Moreover, the motor spins at each frequency for 10 sec in order to get the steady state amplitude of the primary mass. As the motor spins through each frequency, a video is recorded to analyze in the tracker and also a ruler is put next to the apparatus to get the displacements visually.

VII. RESULTS

The raw data collected from the experiment - the steady state amplitude of the primary mass and the corresponding input frequency of the stepper motor - was imported and analyzed using Python. As explained above, there were two different techniques for collecting data. The data generated from the tracker was analyzed using the python script shown in the appendix B. On the other hand, the data collected through visual aid and a ruler was directly plotted without requiring any pre-processing. The corresponding plots for each of the data collection method are shown in figure VII.1 and figure VII.2

Clearly, the uncertainty of the data measured using a ruler is prone to a higher error as the input frequencies increase. This is due to the fact that the primary mass oscillates a faster as the frequency increases and consequently makes it harder to visually determine the change in position. Tracker, on the other hand, provides data at a much higher resolution because of it being automated. However, the video used for tracker is around 290 seconds which equates to about 9000 frames in the tracker which takes about 2 hours to analyze. Accordingly, it becomes harder to analyze in one go. This could have been avoided as explained in section XI.

To verify the results from the tracker data, the experimental results were compared against theoretical plots for various damping ratios as shown in figure VII.3. The equation used for the theoretical lines is shown in equation 1 where various values were plugged in for ζ.

$$x = \frac{\sqrt{1 + [2\zeta\omega/p]^2}}{\sqrt{\left[1 - (\omega/p)^2\right]^2 + [2\zeta\omega/p]^2}} \tag{1}$$

VIII. DISCUSSION

The results shown in figure VII.3 matched reasonably well with the theoretical. To verify that the resonating frequency found matched the theoretical resonating frequency, equation

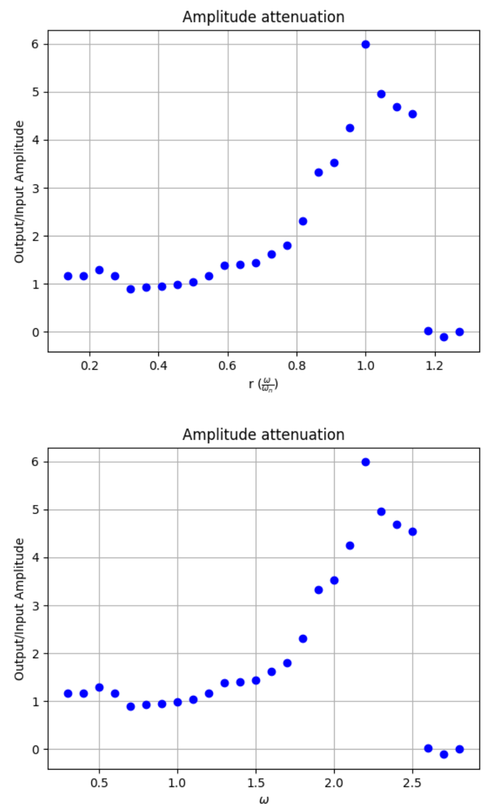

Fig. VII.1. Steady state amplitude vs frequency ratio (top) and input frequency (bottom) measured using tracker.

2 can be used. The mass is known, but the k value for the springs was not given with the purchase of the springs.

To find the k value experimentally, equation 3 was used. A mass was hung from the spring and the displacement from equilibrium was measured. The displacement was found to be 0.05 m for a mass of 0.2 kg. This resulted in a spring constant of 39.5 N/m.

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2}$$

$$F = k\Delta x \tag{3}$$

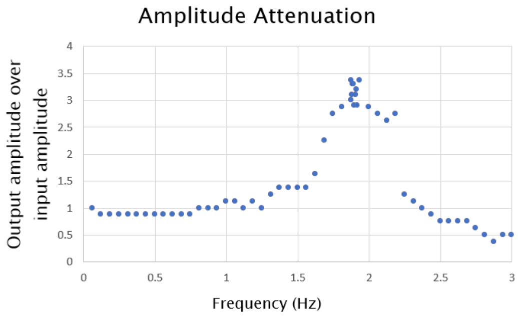

Fig. VII.2. Steady state amplitude vs input frequency measured using a ruler put next to the apparatus.

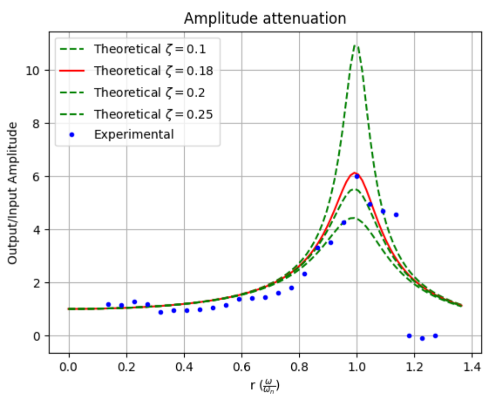

Fig. VII.3. Caption

$$k = \frac{mg}{x}$$
$$k = \frac{(0.2\,\mathrm{kg})(9.80625\,\mathrm{m/s^2})}{(.05\,\mathrm{m})}k \qquad = 39.5\,\mathrm{N/m}$$

It is known that $\frac{\omega}{\omega_n} = 1$. ω_n was determined to be 2.2 Hz from the tracker data. Plugging into equation 2, k was found to be 39.74 N/m.

$$2.2(Hz) \cdot 2\pi(Rad) = \sqrt{\frac{k}{4 \cdot (0.052kg)}}$$
$$k = 39.74N/m$$

This resulted in an Error of 0.6% as shown in equation 4

$$Error = (\frac{39.74 - 39.5}{39.5}) \times 100\% = 0.6\% \tag{4}$$

IX. M1 Application

A core component of approaching experimentation is to be able to identify, test for, and assess system parameters. The experimental apparatuses must also help demonstrate key principles of solid mechanics, thermofluids, and controls.

Specific to educational demonstration, usage demands an apparatus that is accessible, safe, and user-friendly. This apparatus, designed to demonstrate base excitation and, in a more global context, vibration isolation, is appropriate for such demonstrational use.

As an M1 experiment, the characteristic paramaters that are determinable are the input frequency of the stepper motor, as well as the mass, spring, and damping constants. Though these parameters may be indirectly determined as part of the larger experimental procedure, the concept that this rig is best designed for is demonstrating vibration isolation in action.

As such, the most significant parameters that can be identified are the maximum oscillation amplitudes, and the resonance frequency of the determinable system.

The system may be altered across sample groups, so that trends between oscillation amplitude, resonance, and individual system parameters may be identified.

The following is a proposed experimental procedure for use of the apparatus:

1) Begin by unplugging the apparatus and disable power supply.
2) Determine system parameters and apply appropriate masses, spring, and damper to the system.
3) Ensure correct breadboard wiring, especially motor driver wiring. If using additional optical sensors to substitute Tracker use for positional data, ensure proper wiring and connections to breadboard.
4) Determine appropriate frequency range to capture approach to resonance, resonance, and attenuation. This may be completed by testing sample frequencies, or by calculating appropriate amplitude ranges and their frequencies.
5) Plug in the apparatus to an appropriate power supply.
6) Select a frequency, and allow the motor to run.
7) Record the maximum amplitude of oscillation of 20 frequencies, distributed across the predetermined frequency range. This may be completed with use of Tracker to analyze video recording of apparatus motion. If an optical sensor is used instead, allow the apparatus to run until a stable maximum value is found.
8) Plot maxima on an amplitude ratio and frequency ratio graph. This graph may be compared against theoretical predictions, if the system parameters are determined.

X. Conclusions

The analysis of sinusoidal base-excitation behavior showcases an apparatus such as modular rig to be a successful demonstration of vibration isolation. As a visual display, this rig allows for easy identification of resonance and amplitude attenuation, and how these transform as system parameters are set. The close mapping of attenuation amplitude plots against theoretical curves, as well as the convergence of theoretical and experimentally calculated spring constant values promotes use of this model.

Though distance measurement is an appropriate tool to track and characterize system behavior, further development on positional data acquisition would allow for a more self-contained system, avoiding external data processing.

This apparatus may be utilized as an educational demonstration of vibration isolation principles, and may allow users to obtain an intuitive understanding of the methods used to accomplish it.

XI. Recommendations for Future Work

Included on the experimental rig is a time of flight sensor which can be used to determine the output amplitude instead of using tracker or a ruler. The time of flight sensor has a sampling rate of 100 Hz which is not fast enough to accurately plot displacement over time. The onboard microcontroller also has to send signals to the stepper motor which further reduced this sampling rate. Running the motor faster would slow down the sampling from the time of flight sensor and vice versa. This is one of the reasons it was decided not to use the time of flight sensor. Instead, a secondary microcontroller may be used where one drives the motor and the other collects data from the time of flight sensor.

Alternatively, an accelerometer with significantly higher sampling rate can be used with lab view to get more accurate data with a larger number of points. The accelerometer approach is slightly less appealing since by measuring acceleration instead of displacement the ability to recreate the phase plot is removed. However, accelerometers boast a higher sampling rate for a lower cost.

The use of tracker offer's a solution to issues expressed in both the accelerometer approach and the distance sensor approach. With just a few adjustments to the methods used in this paper the tracker data can be significantly more effective.

The main issue with the tracker data was that that tracking point was set to an orange wire wrapped around the masses. Due to a lack of a distinct point to track, the tracking tended to drift up and down along the wire. In an attempt to avoid this issue, the top of the wire was chosen as a tracking point. This would likely have worked great if it wasn't for the background changing color from white to black in the middle of the mass' travel. An image demonstrating this is shown in figure C.1. This caused tracker's auto tracking to fail every time the mass passed this boundary, requiring the point to be re selected every time manually. Eventually the accuracy of the tracking was reduced to allow the software to pass this boundary on its own. This made the drifting problem worse. A better solution would be to secure a small and brightly colored marker onto the middle of the masses to help tracker with its auto tracking. At high frequencies motion blur also caused a problem. A simple solution would be to record video at a higher frame rate, 150 - 200 fps is recommended.

References

[1] Daniel J. Inman. *Engineering Vibration*. Pearson Education Limited, 4th edition, 2014.

Appendix A
Code for running the stepper motor

```
#include <AccelStepper.h>
#include <Arduino.h>

// Assign pins to stepper motor and driver
#define ENABLE_PIN 10
#define STEP_PIN 12
#define DIR_PIN 11

#define STEPS_PER_REV 3200 // Set this value
    according to your stepper motor's
    specifications

float frequency = 0;

// Create stepper object
AccelStepper stepper(AccelStepper::DRIVER,
    STEP_PIN, DIR_PIN);
int max = 0;
int capture = 0;
unsigned long startTime;

void setup() {
  pinMode(ENABLE_PIN, OUTPUT);

  Serial.begin(115200); // Initialize serial
      communication at 9600 bits per second
  // Disable power saving mode on TMC2208 by
      setting ENABLE pin LOW
  digitalWrite(ENABLE_PIN, LOW);
  // Initial settings for stepper motor
  stepper.setAcceleration(2000);
  startTime = millis();
}

  // we get a worst case of 2 samples per cycle
      , so lets capture for 5 seconds for each
      fqz. lets do each fqz from 0 - 45 hz, so
      the total capture time is 45 * 5 = 225
      seconds = 3.75 minutes

void loop()
{
  while(frequency <= 3){
    if( millis() - startTime > 10000){
      Serial.println("Setting␣frequency␣to␣" +
          String(frequency) + "␣Hz");
      // Convert RPM to steps/s
      float speedStepsPerSecond = frequency * (
          STEPS_PER_REV);
      stepper.setMaxSpeed(speedStepsPerSecond);
          // Set max speed for stepper motor
      stepper.setSpeed(speedStepsPerSecond); //
          Set motor speed
      frequency += .1;
      startTime = millis();
    }


    // Run the stepper motor continuously
    stepper.runSpeed();
  }
}
```

APPENDIX B
CODE FOR CREATING THE PLOTS FROM THE TRACKER CSV

```
import pandas as pd
import matplotlib.pyplot as plt

csv_file = 'data.csv' # Replace this with the
    name of your CSV file.
df = pd.read_csv(csv_file)

max_values = []
current_time = 21.727
temp_values = []
time_intervals = []
frequency_intervals = []
frequency = 0.3

for index, row in df.iterrows():
   if row['time'] >= current_time + 10:
      max_values.append(max(temp_values)/.005)
      time_intervals.append(current_time)
      frequency_intervals.append(frequency)
      temp_values = []
      frequency += 0.1
      current_time += 10
   temp_values.append(row['displacement'])

max_values.append(max(temp_values))
time_intervals.append(current_time)
frequency_intervals.append(frequency)

# result_df = pd.DataFrame({'time_interval':
    time_intervals, 'max_displacement':
    max_values})
result_df = pd.DataFrame({'time_interval':
    time_intervals, 'max_displacement':
    max_values, 'frequency':
    frequency_intervals})
result_df.to_csv('output.csv', index=False)


frequency_intervals = [x / 2.2 for x in
    frequency_intervals]
plt.plot(frequency_intervals, max_values, 'bo'
    )
plt.xlabel(r'r␣($\frac{\omega}{\omega_n}$)')
plt.ylabel('Output/Input␣Amplitude')
plt.title('Amplitude␣attenuation')
plt.grid()
plt.show()

# theoretical transmissibility curve
import numpy as np
fqz = np.linspace(0, 3, 100)

naturalFqz = 2.2
r = [x / naturalFqz for x in fqz]
damping = 0.1
OUtOverIn_theoretical= naturalFqz**2 / ((
    naturalFqz**2 - fqz**2)**2 + (2 * damping *
     fqz)**2)**0.5
# dashed line
plt.plot(r, OUtOverIn_theoretical, 'g--',
    label='Theoretical␣$\zeta␣=$' + str(damping
    ))
damping = 0.18
OUtOverIn_theoretical= naturalFqz**2 / ((
    naturalFqz**2 - fqz**2)**2 + (2 * damping *
     fqz)**2)**0.5
plt.plot(r, OUtOverIn_theoretical, 'r', label=
    'Theoretical␣$\zeta␣=$' + str(damping))
damping = 0.2
OUtOverIn_theoretical= naturalFqz**2 / ((
    naturalFqz**2 - fqz**2)**2 + (2 * damping *
     fqz)**2)**0.5
plt.plot(r, OUtOverIn_theoretical, 'g--',
    label='Theoretical␣$\zeta␣=$' + str(damping
    ))
damping = 0.25
OUtOverIn_theoretical= naturalFqz**2 / ((
    naturalFqz**2 - fqz**2)**2 + (2 * damping *
     fqz)**2)**0.5
plt.plot(r, OUtOverIn_theoretical, 'g--',
    label='Theoretical␣$\zeta␣=$' + str(damping
    ))


plt.plot(frequency_intervals, max_values, 'bo'
    , label='Experimental', markersize=3)
plt.xlabel(r'r␣($\frac{\omega}{\omega_n}$)')
plt.ylabel('Output/Input␣Amplitude')
plt.title('Amplitude␣attenuation')
plt.grid()
# put the experimental data on the same plot



plt.legend()
plt.show()
```

APPENDIX C
PICTURE OF TRACKER SETUP

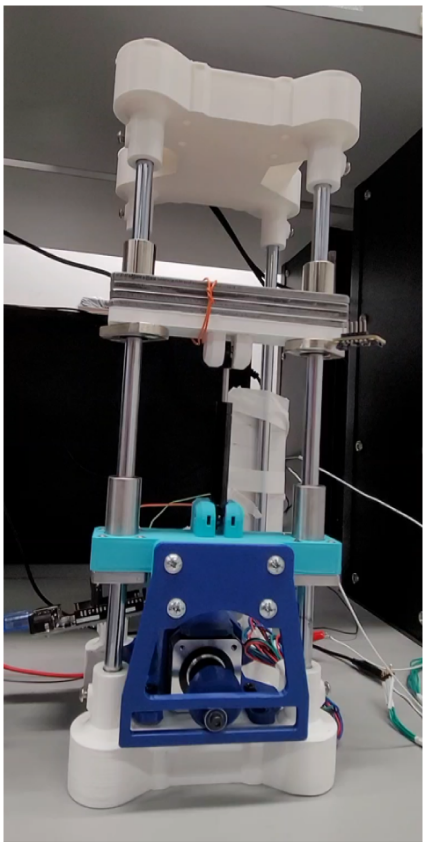

Fig. C.1. Picture of tracker setup